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The Gauge-Invariant Angular Momentum Sum-Rule for the Proton

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Abstract

We give a gauge-invariant treatment of the angular momentum sum-rule for the proton in terms of matrix elements of three gauge-invariant, local composite operators. These matrix elements are decomposed into three independent form factors, one of which is the flavour singlet axial charge. The other two are interpreted as total quark and gluon angular momentum. We further show that the axial charge cancels out of the sum-rule. The general form of the renormalisation mixing of the three operators is written down and also determined to one loop from which the scale dependence and mixing of the form factors is derived. We relate these results to a previous parton model calculation by defining the parton model quantities in terms of the three form factors. We also mention how the form factors can be measured in experiments.

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1 Introduction

In the simplest parton model of the proton, its spin is carried by the spin of the valence quarks in such a way that their overall spin state is that of a spin-1/2 particle. In a QCD-improved parton model, however, such a picture must evolve with the renormalisation scale due to the splitting processes $q \to qg$ etc; at an arbitrary scale, some of the proton spin is then in principle carried by gluons and by strange quarks etc. Furthermore, it was realised some time ago by Ratcliffe [1] that the parton splitting processes generate orbital angular momentum. Thus, if the total spin of the proton is to have a scale-invariant meaning in the context of a parton model, then we must introduce quark and gluon orbital angular momentum components $L_q(x)$ and $L_g(x)$. Together with the usual quark and gluon intrinsic spin components $\Delta q(x)$ and $\Delta g(x)$, which are extracted from polarised Deep Inelastic Scattering (DIS), the sum of their first moments should be scale-invariant.

The splitting functions for these four distributions were calculated by Hoodbhoy, Ji and Tang (HJT) [2] to leading order. The form of the mixing of the first moments[1] under scaling was found to be more general than originally envisaged by Ratcliffe [1]. Importantly, splitting functions do not depend on the choice of gauge used in their calculation. However, it is also claimed by HJT that the first moments of the angular momentum components, Δq, Δg, L_q and L_g, can be represented as forward matrix elements of four composite operators in proton states. Only one of these operators is gauge-invariant. This is the source of many difficulties, some of which are reported in a later paper [5]. As an alternative, if we would define the four components of the proton spin in terms of forward matrix elements of gauge-invariant composite operators, we have a different challenge — there exists only one gauge-invariant operator which can measure total gluon angular momentum (Δg and L_g in the parton model). This means one has to relate four parton model quantities to just three operators. This is discussed in refs. [6, 7]. Moreover, as far as we know, there exists no operator definition of $L_q(x)$ and $L_g(x)$ which scale in the manner of HJT.

In this paper, we study quantities which have a precise operator definition. We discuss the gauge-invariant decomposition of the angular momentum current into three operators, thus restricting ourselves to quantities which can be interpreted as first moments only. We show that the forward matrix elements of the three gauge-invariant operators can be decomposed into just three Lorentz-scalar form factors. One of these form factors is the proton's axial charge which is measured from the flavour singlet component of the first moment of g_1 in polarised DIS. Remarkably, we will show that the axial charge cancels out of the angular momentum sum-rule. We believe that this point has not been emphasised before. Since only two form factors appear in the angular momentum sum rule, we must conclude that the proton spin is only meaningfully decomposed into two pieces. We will interpret the two form factors as total quark and gluon angular momentum. Further, we

[1] Higher moments were calculated in refs. [3, 4].

need not interpret the axial charge as a measure of intrinsic quark spin. Indeed, because of the axial anomaly, the axial charge of the proton is, in the chiral limit, equal to its topological charge, which does not have an obvious interpretation as quark spin since it is defined as the matrix element of a gluon operator. An independent treatment of the anomalous suppression of the first moment of g_1 (the "proton spin" problem) can thus be given in terms of universal topological charge screening [8].

We also derive the renormalisation mixing of the three gauge-invariant operators. We show how, in terms of operator vertices, the axial anomaly affects two of the operators (one is the axial current) so that their sum is free of the anomaly. This is consistent with the fact that the axial charge does not appear in the sum-rule in terms of form factors. We have performed a one-loop calculation of the mixing of the operators.

After presenting our calculations, we then show how they are related to the parton model calculation of HJT. To do this, we shall need to make an identification between the three form factors including the axial charge and the four parton model quantities. This is necessarily a Lorentz frame dependent statement, but which reproduces their results.

Recently, it was proposed [7, 10] that the two new form factors could be measured in hard, exclusive processes $\gamma^* p \to Xp$ such as Deeply Virtual Compton Scattering (DVCS) where $X = \gamma$. A section reviewing this possibility is included.

The paper is organised as follows: In section 2 we review the decomposition of the conserved angular momentum current (AMC) into spin and orbital pieces, explaining why the gluon piece cannot be decomposed in this way gauge-invariantly. We include the gauge-fixing terms from the Lagrangian. Our main results are contained in sections 3 and 4: In section 3 we decompose the forward matrix elements of the three operators in the gauge-invariant decomposition of the AMC into scalar form factors; section 4 contains the results of a one-loop calculation of the divergent pieces of the three operators, from which are derived the evolution and mixing of the form factors. In section 5 we discuss how to relate the form factors to parton model quantities, and how the result of HJT is obtained. In section 6 we review how the two new form factors can be extracted from a class of hard, exclusive processes such as Deeply Virtual Compton Scattering (DVCS). Finally, section 7 reviews attempts made in the literature to identify angular momentum components of the proton to matrix elements of non-gauge-invariant operators, highlighting the problems with such treatments.

2 Decomposition of Angular Momentum Current

In this section, we summarise the construction of a conserved angular momentum current (AMC) $M^{\mu\nu\lambda}$ from the QCD lagrangian. As with the stress-energy tensor $T^{\mu\nu}$, we have the freedom to redefine $M^{\mu\nu\lambda}$ by adding on divergence pieces such that $M^{\mu\nu\lambda}$ is still conserved.

As is discussed in ref. [6], it is possible in this way to separate the quark part of $M^{\mu\nu\lambda}$ into spin and orbital angular momentum (OAM) pieces. However, it is not possible to do the same for the gluon term in a gauge-invariant fashion. The reason is that the gauge field's spin and space-time indices coincide. Below, we extend the discussion in ref. [6] to include extra terms coming from gauge-fixing terms in covariant gauges and show that the decomposition of the AMC will depend on the details of the gauge-fixing procedure, so that even the ghost fields carry OAM. We will conclude from this that it is not sensible to define observables except in terms of the gauge invariant decomposition, even though this means we are left with only three operators measuring observable quantities instead of four.

We begin with the divergence-free, symmetric stress-energy tensor $T^{\mu\nu}$. From it we can construct $M^{\mu\nu\lambda}$:

$$M^{\mu\nu\lambda} = x^{[\nu}T^{\lambda]\mu}, \tag{1}$$

where the square brackets denote antisymmetrisation.[2] This is conserved ($\partial_\mu M^{\mu\nu\lambda} = 0$) provided that $T^{\mu\nu}$ is symmetric. It is always possible to redefine $M^{\mu\nu\lambda}$ by adding the divergence of a function $X^{\mu\alpha\nu\lambda}$:

$$M'^{\mu\nu\lambda} = M^{\mu\nu\lambda} + \partial_\alpha X^{\mu\alpha\nu\lambda}. \tag{2}$$

M' is also conserved provided X is antisymmetric under $\mu \leftrightarrow \alpha$ as well as under $\nu \leftrightarrow \lambda$.[3]

If we now consider the covariantly gauge-fixed, BRST-invariant QCD lagrangian,

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{gi}} + \mathcal{L}_{\text{gf}}, \\ \mathcal{L}_{\text{gi}} &= \overline{\psi} i \overleftrightarrow{\not{D}} \psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \\ \mathcal{L}_{\text{gf}} &= -i\partial^\mu \overline{c} D_\mu c - \partial^\mu B A_\mu + \frac{\alpha}{2}B^2, \end{aligned} \tag{3}$$

the corresponding symmetric stress-energy tensor is

$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}_{\text{gi}} + T^{\mu\nu}_{\text{gf}}, \\ T^{\mu\nu}_{\text{gi}} &= \frac{1}{2}\overline{\psi}\gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} \psi + F^{\mu\alpha}F_\alpha{}^\nu - g^{\mu\nu}\mathcal{L}_{\text{gi}}, \\ T^{\mu\nu}_{\text{gf}} &= -i\partial^{\{\mu}\overline{c}D^{\nu\}}c - \partial^{\{\mu}BA^{\nu\}} - g^{\mu\nu}\mathcal{L}_{\text{gf}}. \end{aligned} \tag{4}$$

Writing $M^{\mu\nu\lambda}_{\text{gi}} = x^{[\nu}T^{\lambda]\mu}_{\text{gi}}$, one can show that

$$\begin{aligned} M^{\mu\nu\lambda}_{\text{gi}} &= \frac{1}{2}\epsilon^{\mu\nu\lambda\alpha}\overline{\psi}\gamma_\alpha\gamma^5\psi + \overline{\psi}\gamma^\mu x^{[\nu} i \overleftrightarrow{D}^{\lambda]} \psi + x^{[\nu}F^{\lambda]\alpha}F_\alpha{}^\mu - x^{[\nu}g^{\lambda]\mu}\mathcal{L}_{\text{gi}} \\ &- \frac{1}{4}\partial_\alpha\left[x^{[\nu}\epsilon^{\lambda]\mu\alpha\beta}\overline{\psi}\gamma_\beta\gamma^5\psi\right] + \text{EOM}. \end{aligned} \tag{5}$$

[2]Our convention is that $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$, while $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$.

[3]Note that a transformation $T'^{\mu\nu} = T^{\mu\nu} + \partial_\alpha X^{\mu\nu\alpha}$ does not correspond to a transformation of $M^{\mu\nu\lambda}$.

Defining M' so as to cancel the divergence and equation of motion (EOM) pieces, we have

$$\begin{aligned} M'^{\mu\nu\lambda} &= M'^{\mu\nu\lambda}_{\rm gi} + M'^{\mu\nu\lambda}_{\rm gf}, \\ M'^{\mu\nu\lambda}_{\rm gi} &= O_1^{\mu\nu\lambda} + O_2^{\mu[\lambda}x^{\nu]} + O_3^{\mu[\lambda}x^{\nu]} - x^{[\nu}g^{\lambda]\mu}\mathcal{L}_{\rm gi}, \\ M'^{\mu\nu\lambda}_{\rm gf} &= x^{[\nu}T_{\rm gf}^{\lambda]\mu} \end{aligned} \tag{6}$$

with

$$\begin{aligned} O_1^{\mu\nu\lambda} &= \frac{1}{2}\epsilon^{\mu\nu\lambda\alpha}\overline{\psi}\gamma_\alpha\gamma^5\psi, \\ O_2^{\mu\nu} &= \overline{\psi}\gamma^\mu i \overleftrightarrow{D}^\nu \psi, \\ O_3^{\mu\nu} &= F^{\mu\alpha}F_\alpha{}^\nu. \end{aligned} \tag{7}$$

We have in this way split the quark piece into what appears to be a spin piece, the axial current O_1, and an OAM piece xO_2, while xO_3 is the total gluon angular momentum, both spin and OAM. All the terms in $M'_{\rm gi}$ are gauge-invariant. Matrix elements of $M'_{\rm gf}$ vanish since it is a BRST variation of another operator. Also, forward matrix elements of the term propertional to $x^{[\nu}g^{\lambda]\mu}$ vanish; this will be shown in the next section. It is this decomposition which we will use to derive the scaling of the components of a nucleon's spin: It ensures that all quantities are explicitly gauge-invariant.

Now consider what we get when we try to split the gluon component O_3 into spin and OAM parts. Again, one can show that

$$\begin{aligned} M'^{\mu\nu\lambda} &= \frac{1}{2}\epsilon^{\mu\nu\lambda\alpha}\overline{\psi}\gamma_\alpha\gamma^5\psi + i\overline{\psi}\gamma^\mu x^{[\nu}\partial^{\lambda]}\psi \\ &- F^{\mu[\nu}A^{\lambda]} - F^{\mu\alpha}x^{[\nu}\partial^{\lambda]}A_\alpha \\ &- A^\mu x^{[\nu}\partial^{\lambda]}B - i\partial^\mu\overline{c}x^{[\nu}\partial^{\lambda]}c - ix^{[\nu}\partial^{\lambda]}\overline{c}D^\mu c - x^{[\nu}g^{\lambda]\mu}\mathcal{L} \\ &+ \partial_\alpha\left[F^{\mu\alpha}x^{[\nu}A^{\lambda]}\right] + \text{EOM}. \end{aligned} \tag{8}$$

We can redefine $M' \to M''$ again by dropping the divergence and EOM terms. The new decomposition coincides with that of the canonical angular momentum current: Defining the rotation generator J^{ij},

$$J^{ij} = \int (\mathrm{d}^3x)_\mu M^{\mu ij}, \tag{9}$$

if we choose the integral to be over 3-dimensional space at a given time, we have

$$\begin{aligned} J^{ij}_{(0)} &= \int \mathrm{d}^3x M^{0ij} \\ &= \int \mathrm{d}^3x \left[\psi^\dagger\sigma^{ij}\psi + i\psi^\dagger x^{[i}\partial^{j]}\psi - E^{[i}A^{j]} + E^k x^{[i}\partial^{j]}A^k \right. \\ &- \left. A^0 x^{[i}\partial^{j]}B - i\partial^0\overline{c}x^{[i}\partial^{j]}c - ix^{[i}\partial^{j]}\overline{c}D^0 c\right], \end{aligned} \tag{10}$$

with $E^i = F^{0i}$ and $\sigma^{ij} = \frac{i}{4}[\gamma^i, \gamma^j]$. This may be written

$$J^{ij}_{(0)} = \sum_\phi \int \mathrm{d}^3x\; i\pi(\phi_b)(\delta^{ij})_{ba}\phi_a, \tag{11}$$

where ϕ_a are the canonical fields of the Kugo-Ojima canonical formulation of non-Abelian gauge theory [11], $\pi(\phi_a)$ are their conjugate momenta, and δ^{ij} are their variations under a spatial rotation.[4]

One might be tempted to take the first four terms in eq. (8) and identify their matrix elements with quark spin and OAM and gluon spin and OAM respectively. There are good reasons for not doing so:

1. The other terms in eq. (8) involving the gauge-fixing fields $c, \overline{c}, B$ are no longer a BRST invariant combination; the sum of their matrix elements cannot be expected to vanish. This means that we would have to talk about ghost OAM etc. in order to have a basis of operators which is closed upon renormalisation.

2. The mixing amongst quark and gluon spin and OAM will depend on the gauge parameter, as observed in ref. [2]. The observables whose scaling we wish to derive, on the other hand, are of course gauge invariant.

3 Role of the Axial Charge

We now develop further the gauge-invariant decomposition of the angular momentum current (AMC) in eq. (6). First, we decompose the forward matrix elements of the operators in the AMC into three Lorentz scalar form factors. One of these, the axial charge, appears twice with opposite sign, and so cancels out of the angular momentum sum-rule. This allows us to conclude that firstly the axial charge does not form part of the angular momentum sum rule and secondly that we do not need to interpret the axial charge as a measure of parton spin. The other two form factors have been discussed before [6, 7], however the relationship of these to the axial anomaly has not been emphasised. Furthermore, if the axial charge is to decouple from the sum rule in this way, we should be able to show this is reflected in the renormalisation of the AMC. To our knowledge, an explicit derivation of the mixing of the three gauge invariant operators appearing in the AMC has not yet appeared. Therefore, below we derive the general form of the mixing. In the next section we calculate the mixing explicitly to one loop.

The axial charge a_0 is defined to be a Lorentz scalar form factor, or reduced matrix element:

$$\left\langle p, s \left| O_1^{\mu\nu\lambda} \right| p, s \right\rangle = M \epsilon^{\mu\nu\lambda\rho} s_\rho a_0, \tag{12}$$

We should be able to define all our observables in terms of such scalar functions, so let us now decompose forward matrix elements of all the remaining operators in the AMC (6)

[4]Each term in $J^{ij}_{(0)}$ satisfies the generator algebra for rotations. Note, however, that when the operators are renormalised, this is no longer true. Indeed, this even happens for the first term, the axial current, due to the axial anomaly.

except the divergence and the EOM, i.e. $O_2^{\mu[\lambda}x^{\nu]}$, $O_3^{\mu[\lambda}x^{\nu]}$ and $g^{\mu[\lambda}x^{\nu]}\mathcal{L}$. To do this, we need to define the forward matrix element of an operator involving a factor of the coordinate x^μ. This is done in Appendix A. (See refs. [2, 3].) The result is that if $B^{\mu\nu}(x)$ is a local composite operator, then the forward matrix element of $x^{[i}B^{j]0}(x)$ is

$$\Phi^{-1}(p)\int \mathrm{d}^3x \left\langle p \left| x^{[i}B^{j]0}(x) \right| \Phi \right\rangle = -i\frac{\partial}{\partial \Delta_{[i}} \left\langle p \left| B^{j]0}(0) \right| p' \right\rangle \bigg|_{p'=p}, \tag{13}$$

where $\Phi(p)$ is a wavepacket with no azimuthal dependence about the k-direction and where $P = \frac{1}{2}(p+p')$ is held constant in the partial derivative w.r.t. $\Delta = p - p'$. Before using this expression, we decompose the off-forward matrix elements of $g^{\mu\nu}\mathcal{L}$, $O_2^{\mu\nu}$, $O_3^{\mu\nu}$ and

$$O_4^{\mu\nu} = \frac{1}{2}\overline{\psi}\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}\psi \tag{14}$$

into scalar form factors. We write down only those terms which are linear in Δ and which are even in P; the latter constraint comes from symmetry under crossing, $\Delta \to \Delta$ and $P \to -P$. The Lorentz structures must also have even parity:

$$\begin{aligned}
\langle p,s\,| g^{\mu\nu}\mathcal{L} |\, p',s\rangle &= C_{\mathcal{L}}(\Delta^2)M^2 g^{\mu\nu}, \\
\langle p,s\,| O_2^{\mu\nu}(0) |\, p',s\rangle &= A_q(\Delta^2)P^\mu P^\nu + \frac{B_q(\Delta^2)}{2M}P^{\{\mu}\epsilon^{\nu\}\alpha\beta\sigma}i\Delta_\alpha P_\beta s_\sigma + C_q(\Delta^2)M^2 g^{\mu\nu} \\
&+ \frac{\widetilde{B}_q(\Delta^2)}{2M}P^{[\mu}\epsilon^{\nu]\alpha\beta\sigma}i\Delta_\alpha P_\beta s_\sigma + MD_q(\Delta^2)\epsilon^{\mu\nu\alpha\beta}i\Delta_\alpha s_\beta, \\
\langle p,s\,| O_3^{\mu\nu}(0) |\, p',s\rangle &= A_g(\Delta^2)P^\mu P^\nu + \frac{B_g(\Delta^2)}{2M}P^{\{\mu}\epsilon^{\nu\}\alpha\beta\sigma}i\Delta_\alpha P_\beta s_\sigma + C_g(\Delta^2)M^2 g^{\mu\nu}, \\
\langle p,s\,| O_4^{\mu\nu}(0) |\, p',s\rangle &= A_q(\Delta^2)P^\mu P^\nu + \frac{B_q(\Delta^2)}{2M}P^{\{\mu}\epsilon^{\nu\}\alpha\beta\sigma}i\Delta_\alpha P_\beta s_\sigma + C_q(\Delta^2)M^2 g^{\mu\nu}.
\end{aligned} \tag{15}$$

Notice that the symmetric part of O_4 is identical to that of O_2. Substituting these in the r.h.s. of (13) we get

$$\begin{aligned}
-i\frac{\partial}{\partial \Delta_\nu}\left\langle p,s \left| g^{\mu\lambda}\mathcal{L}(0) \right| p',s \right\rangle \bigg|_{p=p'} &= 0, \\
-i\frac{\partial}{\partial \Delta_\nu}\left\langle p,s \left| O_2^{\mu\lambda}(0) \right| p',s \right\rangle \bigg|_{p=p'} &= \frac{B_q(0)}{2M}p^{\{\mu}\epsilon^{\lambda\}\nu\beta\sigma}p_\beta s_\sigma, \\
&+ \frac{\widetilde{B}_q(0)}{2M}p^{[\mu}\epsilon^{\lambda]\nu\beta\sigma}p_\beta s_\sigma - MD_q(0)\epsilon^{\mu\nu\lambda\beta}s_\beta, \\
-i\frac{\partial}{\partial \Delta_\nu}\left\langle p,s \left| O_3^{\mu\lambda}(0) \right| p',s \right\rangle \bigg|_{p=p'} &= \frac{B_g(0)}{2M}p^{\{\mu}\epsilon^{\lambda\}\nu\beta\sigma}p_\beta s_\sigma, \\
-i\frac{\partial}{\partial \Delta_\nu}\left\langle p,s \left| O_4^{\mu\lambda}(0) \right| p',s \right\rangle \bigg|_{p=p'} &= \frac{B_q(0)}{2M}p^{\{\mu}\epsilon^{\lambda\}\nu\beta\sigma}p_\beta s_\sigma.
\end{aligned} \tag{16}$$

There are further constraints on the five remaining form factors: We showed in deriving eq. (5) that

$$O_4^{\mu[\lambda}x^{\nu]} = O_1^{\mu\nu\lambda} + O_2^{\mu[\lambda}x^{\nu]} + \text{divergence} + \text{EOM}, \tag{17}$$

It is shown in Appendix A that the forward matrix element of the divergence term vanishes. It follows from eqs. (12,16,17) that

$$\begin{aligned} \widetilde{B}_q(0) &= 0, \\ 2D_q(0) &= a_0. \end{aligned} \tag{18}$$

Eqs. (12) and (16) therefore become

$$\begin{aligned} \left\langle p, s \left| O_1^{\mu\nu\lambda}(0) \right| p', s \right\rangle &= M a_0 \epsilon^{\mu\nu\lambda\beta} s_\beta, \\ -i\frac{\partial}{\partial \Delta_\nu} \left\langle p, s \left| O_2^{\mu\lambda}(0) \right| p', s \right\rangle \Big|_{p=p'} - (\nu \leftrightarrow \lambda) &= \frac{B_q(0)}{2M} p^{\{\mu}\epsilon^{\lambda\}\nu\beta\sigma} p_\beta s_\sigma - (\nu \leftrightarrow \lambda) - M a_0 \epsilon^{\mu\nu\lambda\beta} s_\beta, \\ -i\frac{\partial}{\partial \Delta_\nu} \left\langle p, s \left| O_3^{\mu\lambda}(0) \right| p', s \right\rangle \Big|_{p=p'} - (\nu \leftrightarrow \lambda) &= \frac{B_g(0)}{2M} p^{\{\mu}\epsilon^{\lambda\}\nu\beta\sigma} p_\beta s_\sigma - (\nu \leftrightarrow \lambda). \end{aligned} \tag{19}$$

The decomposition in eqs. (19) is one of the main points of this paper: There can be no physically meaningful further decomposition of the proton spin in terms of forward matrix elements, since we only measure gauge-invariant quantities. Moreover, by adding the three equations together, one can see that the axial charge a_0 cancels from the sum-rule, so that only $B_{q/g}(0)$ appear in it. Thus, a_0 does not necessarily have to be interpreted as a measure of parton intrinsic spin. The best interpretation that one can give for $B_{q/g}(0)$ are the total angular momentum of the quarks and gluons $J_{q/g}$ respectively:

$$\begin{aligned} B_q(0) &= J_q, \\ B_g(0) &= J_g. \end{aligned} \tag{20}$$

We refrain from decomposing $J_{q/g}$ further into spin plus orbital, since it is not clear that it is meaningful to do so. We shall discuss this further in section 5. The sum-rule

$$B_q(0) + B_g(0) = J_q + J_g = \frac{1}{2} \tag{21}$$

is assured by the non-renormalisation of the stress-energy tensor. This holds in spite of the presence of the axial anomaly because in eq. (19) it cancels out of the angular momentum sum-rule.

In the next section, we calculate the scaling of the operators $O_{1\to 3}$ to one loop. Because this is done in the language of operator vertices, as in the OPE analysis of DIS, it is important to show how the above conclusions translate into the language of mixing of composite operators. We shall argue that the axial anomaly affects not only the renormalisation of O_1 but also of xO_2. To do this, we appeal firstly to some general properties

of local gauge-invariant operators, and secondly to the non-renormalisation of the AMC, which we have verified to one loop in section 4. We can thus write down the general form of the mixing of the operators of the AMC.

First, we use the property that operators of the form xO_i and O_a when inserted in forward matrix elements mix with a block triangular matrix:

$$\left(\begin{array}{c} O_a \\ xO_i \end{array} \right)_R = \left(\begin{array}{cc} Z_{ab}^{-1} & 0 \\ Z_{ib}^{-1} & Z_{ij}^{-1} \end{array} \right) \left(\begin{array}{c} O_b \\ xO_j \end{array} \right)_B . \tag{22}$$

This is assured because local, gauge-invariant composite operators with no factors of the co-ordinate x^μ only mix with other such operators; the top right block of the matrix is therefore zero. Second, we note that because $O_3^{\mu\nu}$ is a symmetric tensor, it can only mix with the symmetric operators $O_3^{\mu\nu}$ and $O_4^{\mu\nu}$. From eq. (17), it then follows that $O_3^{\mu[\lambda}x^{\nu]}$ can only mix with $O_3^{\mu[\lambda}x^{\nu]}$ and $O_1^{\mu\nu\lambda} + O_2^{\mu[\lambda}x^{\nu]}$, when inserted in forward matrix elements. Finally, given that the AMC is not renormalised, the columns of the mixing matrix should add up to one. With these contraints, the most general form of the matrix is as follows:

$$\left(\begin{array}{c} O_1^{\mu\nu\lambda} \\ O_2^{\mu[\lambda}x^{\nu]} \\ O_3^{\mu[\lambda}x^{\nu]} \end{array} \right)_B = \left(\begin{array}{ccc} 1+X & 0 & 0 \\ Z-X & 1+Z & -Y \\ -Z & -Z & 1+Y \end{array} \right) \left(\begin{array}{c} O_1^{\mu\nu\lambda} \\ O_2^{\mu[\lambda}x^{\nu]} \\ O_3^{\mu[\lambda}x^{\nu]} \end{array} \right)_R , \tag{23}$$

where $X = O(\alpha_s^2)$, $Y = O(\alpha_s)$ and $Z = O(\alpha_s)$. The piece X is due to the anomaly, which appears at two loops. It cancels between O_1 and xO_2 in the AMC. In terms of the form factors, this mixing matrix becomes

$$\left(\begin{array}{c} a_0 \\ B_q(0) \\ B_g(0) \end{array} \right)_B = \left(\begin{array}{ccc} 1+X & 0 & 0 \\ 0 & 1+Z & -Y \\ 0 & -Z & 1+Y \end{array} \right) \left(\begin{array}{c} a_0 \\ B_q(0) \\ B_g(0) \end{array} \right)_R . \tag{24}$$

In the above, we have taken the AMC current not to be renormalised. This we verify explicitly to one loop in the next section.

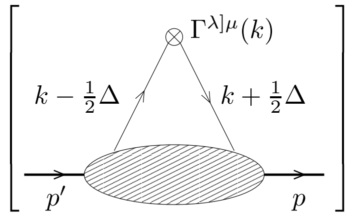
4 One-Loop Evolution

In this part, we present the mixing amongst the operators O_1, O_2, O_3 to leading order. First, we discuss the technicalities of operator mixing. The theory of mixing of gauge-invariant composite operators such as O_1 is well understood. The extension to operators with a factor of x^μ is straightforward. In particular, we show how operators of the form xO_i can mix with operators of the form O_a.

Suppose we have a local composite operator $B^{\mu\nu}(x)$ which mixes with a total derivative operator $\partial_\alpha A^{\mu\nu\alpha}(x)$. The forward matrix element of the total derivative operator would

vanish. However, an operator $x^\lambda B^{\mu\nu}(x)$ would correspondingly mix with $x^\lambda \partial_\alpha A^{\mu\nu\alpha}(x)$ which is not a total derivative, but which equals $\partial_\alpha(x^\lambda A^{\mu\nu\alpha}(x)) - A^{\mu\nu\lambda}(x)$. The first term has vanishing forward matrix element while the second is a local, composite operator with no factor x^μ so gives an off-diagonal term in the mixing matrix. To see this explicitly, we represent the forward matrix element of $x^{[\nu} B^{\lambda]\mu}(x)$ as (See Appendix A.)

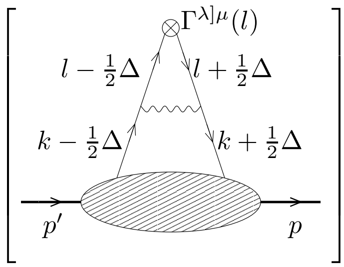
$$\lim_{p'\to p} -i\frac{\partial}{\partial \Delta_{[\nu}}$$



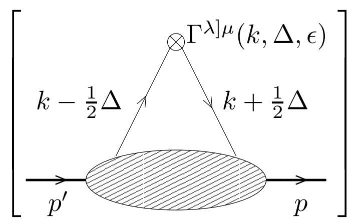
$$, \qquad (25)$$

where $\Gamma^{\mu\nu}$ is the momentum-space vertex associated with the operator $B^{\mu\nu}$, and the shaded blob is a four-point function, 1PI w.r.t. the p and p' legs. As an example, consider the UV divergence of a loop correction to $\Gamma^{\mu\nu}$:

$$-i\frac{\partial}{\partial \Delta_{[\nu}}$$



$$\xrightarrow{\text{large } l^2} -i\frac{\partial}{\partial \Delta_{[\nu}}$$



$$. \qquad (26)$$

The divergence piece takes the form, in dimensional regularisation, of $1/\epsilon$ pole pieces:

$$\Gamma^{\mu\nu}(k,\Delta,\epsilon) = \Gamma_0^{\mu\nu}(k,\epsilon) + i\Delta_\alpha \Gamma_1^{\mu\nu\alpha}(k,\epsilon) + O(\Delta^2). \qquad (27)$$

The $O(\Delta)$ pieces, $\Delta_\alpha \Gamma_1^{\mu\nu\alpha}$, correspond to mixing with a total derivative operator $\partial_\alpha A^{\mu\nu\alpha}$. The Δ differentiation acts on the blob, and then on the pole pieces. The first corresponds to mixing with another operator of the form xO_i, while the second gives

$$-i\frac{\partial}{\partial \Delta_{[\nu}} \Gamma^{\lambda]\mu}(k,\Delta,\epsilon) = \Gamma_1^{\lambda\mu\nu}(k,\epsilon) - (\nu \leftrightarrow \lambda) + O(\Delta). \qquad (28)$$

Thus, when considering the forward matrix elements, the mixing of an operator $B^{\mu\nu}$ with a total derivative operator $\partial_\alpha A^{\mu\nu\alpha}$ corresponds to mixing of $B^{\mu[\lambda}x^{\nu]}$ with $A^{\mu[\lambda\nu]}$.

Now we give the one-loop mixing of the operators in the gauge-invariant decomposition of the AMC. The basis of operators required is the following:

$$\begin{aligned} O_1^{\mu\nu\lambda} &= \frac{1}{2}\epsilon^{\mu\nu\lambda\alpha}\overline{\psi}\gamma_\alpha\gamma^5\psi, \\ O_2^{\mu\nu} &= \overline{\psi}\gamma^\mu i \overleftrightarrow{D}^\nu \psi, \\ O_3^{\mu\nu} &= F^{\mu\alpha}F_\alpha{}^\nu, \\ O_4^{\mu\nu} &= \frac{1}{2}\overline{\psi}\gamma^{\{\mu} i \overleftrightarrow{D}^{\nu\}} \psi. \end{aligned} \qquad (29)$$

These operators also mix with $g^{\mu\nu}O_5$ and $g^{\mu\nu}E_1$, where

$$\begin{aligned} O_5 &= F^{\alpha\beta}F_{\alpha\beta}, \\ E_1 &= i\overline{\psi}\overleftrightarrow{\not{D}}\psi, \end{aligned} \tag{30}$$

but we shall not consider these since not only are they projected out in the definition of J^{ij} in eq. (9), but also the first derivatives of their matrix elements vanish in the forward limit. We do not require operators of lower dimension, since we are neglecting quark masses. Note that we require not just $O_{1\to 3}$ but also O_4, which is O_2 with its indices symmetrised. The fact that O_2 and O_3 will mix with O_4 does not spoil the non-renormalisation of the g.i. decomposition because of eq. (17).

Inserting the operators in the Green functions with two gluon or two quark legs, we calculate the pole pieces to one loop. Details are to be found in Appendix B. We find that the matrix Z_{ij} required to renormalise the bare operators is

$$\begin{aligned} (O_i)_B &= Z_{ij}(O_j)_R, \\ \begin{pmatrix} \partial_\lambda O_1^{\mu\nu\lambda} \\ O_2^{\mu\nu} \\ O_3^{\mu\nu} \\ O_4^{\mu\nu} \end{pmatrix}_B &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{2}{3}n_f\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} & -\frac{8}{3}C_F\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} \\ 0 & 0 & 1-\frac{2}{3}n_f\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} & \frac{8}{3}C_F\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} \\ 0 & 0 & \frac{2}{3}n_f\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} & 1-\frac{8}{3}C_F\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} \end{pmatrix} \begin{pmatrix} \partial_\lambda O_1^{\mu\nu\lambda} \\ O_2^{\mu\nu} \\ O_3^{\mu\nu} \\ O_4^{\mu\nu} \end{pmatrix}_R . \end{aligned} \tag{31}$$

The angular momentum current is not renormalised to this order. Note also that the counterterms for O_2 are the same as those for O_4. Thus, the renormalisation of xO_2 and xO_3 follows that for the stress-energy tensor $T^{\mu\nu}$. At the two-loop level, we anticipate that this behaviour will be modified by the axial anomaly, where O_1 mixes with itself only [14]:

$$(O_1^{\mu\nu\lambda})_B = Z_{11}(O_1^{\mu\nu\lambda})_R. \tag{32}$$

If the AM current is not to be renormalised, then xO_2 will have to mix with O_1.

Using eq. (17), when the operators are inserted in forward matrix elements, eq. (31) becomes

$$\begin{pmatrix} O_1^{\mu\nu\lambda} \\ O_2^{\mu[\nu}x^{\lambda]} \\ O_3^{\mu[\nu}x^{\lambda]} \end{pmatrix}_B = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{8}{3}C_F\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} & 1-\frac{8}{3}C_F\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} & \frac{2}{3}n_f\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} \\ \frac{8}{3}C_F\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} & \frac{8}{3}C_F\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} & 1-\frac{2}{3}n_f\frac{\alpha_s}{4\pi}\frac{1}{\epsilon} \end{pmatrix} \begin{pmatrix} O_1^{\mu\nu\lambda} \\ O_2^{\mu[\nu}x^{\lambda]} \\ O_3^{\mu[\nu}x^{\lambda]} \end{pmatrix}_R . \tag{33}$$

If we now take the forward matrix elements of O_1, xO_2 and xO_3 as defined by eq. (13) we obtain, to $O(\alpha_s)$,

$$\begin{aligned} \frac{d}{dt}a_0 &= 0 \\ \frac{d}{dt}B_q &= \frac{\alpha_s}{4\pi}\left[-\frac{8}{3}C_F B_q + \frac{2}{3}n_f B_g\right] \\ \frac{d}{dt}B_g &= \frac{\alpha_s}{4\pi}\left[\frac{8}{3}C_F B_q - \frac{2}{3}n_f B_g\right]. \end{aligned} \tag{34}$$

5 The Parton Model Decomposition

Since the angular momentum sum rule only involves two form factors and is independent of the axial charge and the anomaly associated with it, the best decomposition of the proton spin that we can have is

$$J_q + J_g = \frac{1}{2}. \tag{35}$$

In the context of a parton model, we would like to be able to write

$$\frac{1}{2}\Delta q + \Delta g + L_q + L_g = \frac{1}{2}, \tag{36}$$

so that

$$\begin{aligned} J_q &= \frac{1}{2}\Delta q + L_q, \\ J_g &= \Delta g + L_g. \end{aligned} \tag{37}$$

This is not a natural thing to do since we would like to identify Δq with the axial charge, which multiplies a different Lorentz structure from $J_{q/g}$ in the form factor decomposition. In other words, such a decomposition is dependent on the Lorentz frame.

Nevertheless, to make contact with the results of HJT [2], we also relate the three form factors in terms of parton model quantities. As remarked above, the central problem is to relate four observables, Δq, Δg, L_q and L_g to just three form factors, not least because we can imagine each scaling in a distinct way. We will argue, however, that because it is possible to define Δq to be scale-independent, then we may determine the scaling of Δg, L_q and L_g gauge-invariantly in terms of the scaling of the three gauge-invariant operators in eq. (6). The problem is the same as that in polarised DIS, where there are two observables, Δq and Δg to relate to just one form factor, a^0. Below, we briefly review polarised DIS and the role of the axial anomaly, and how Δq can be constrained to be scale-independent in a certain class of renormalisation schemes. We then identify form factors with parton model quantities, and rederive the result of HJT [2] for the mixing of Δq, Δg, L_q and L_g. Thus we will have shown that an operator definition of these quantities does in fact lead to the HJT mixing.

Let us begin with the first moment Γ_1 of the polarised structure function $g_1(x, Q^2)$ as measured in polarised DIS. For large Q^2, Γ_1 factorises in the following way:

$$\begin{aligned} \Gamma_1 = \int_0^1 dx\, g_1(x, Q^2) &= \frac{\langle e^2 \rangle}{2}\left[C^1_{\mathrm{NS}}(\alpha_s)\Delta q_{\mathrm{NS}} + C^1_{\mathrm{S}}(\alpha_s)\Delta q + 2n_f C^1_g(\alpha_s)\Delta g\right], \\ \Delta q_{\mathrm{NS}} &= \frac{1}{n_f}\sum_{i=1}^{n_f}\left(\frac{e_i^2}{\langle e^2\rangle} - 1\right)(\Delta q_i + \Delta\overline{q}_i), \\ \Delta q &= \sum_{i=1}^{n_f}(\Delta q_i + \Delta\overline{q}_i), \end{aligned} \tag{38}$$

where $\langle e^2 \rangle = \frac{1}{n_f} \sum_i e_i^2$, Δq_i, $\Delta \overline{q}_i$ and Δg are the first moments of the polarised quark, antiquark and gluon parton distribution functions (PDFs) respectively, and $C^1(\alpha_s)$ are the first moments of the co-efficient functions. Both the co-efficient functions and the PDFs are scheme dependent. At leading order, parton amplitude calculations show that the scaling of Δq and Δg are given by

$$\frac{d}{dt} \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} = \frac{\alpha_s}{4\pi} \begin{pmatrix} 0 & 0 \\ 3C_F & \beta_0 \end{pmatrix} \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix}, \tag{39}$$

where $C_F = \frac{4}{3}$ and $\beta_0 = 11 - \frac{2}{3} n_f$. The eigenvectors of this evolution are Δq, which does not evolve, and $\Delta q - 2n_f \frac{\alpha_s}{4\pi} \Delta g$. It then follows that there should exist a class of renormalisation schemes, the Adler-Bardeen (AB) schemes [9], where these two eigenvectors are the same to all orders. In such a scheme, it is the second of the eigenvectors which may then appear in the factorisation in eq. (38): By contrast, in the $\overline{\mathrm{MS}}$ scheme, the gluon co-efficient $C_g^1(\alpha_s)$ vanishes to $O(\alpha_s^2)$, so that the only flavour singlet piece in Γ_1 is the contribution from Δq. The factorisation can also be expressed in terms of the operator product expansion (OPE). Since the only flavour singlet operator with the right dimension is the axial current,

$$\left\langle p, s \left| O_1^{\mu\nu\lambda} \right| p, s \right\rangle = M \epsilon^{\mu\nu\lambda\rho} s_\rho a_0(t), \tag{40}$$

we have, for the $\overline{\mathrm{MS}}$ scheme, $a^0(t) = \Delta q(t)$. That is, Δg does not contribute, while $\Delta q(t)$ depends on the scale t because of the axial anomaly. However, in the class of AB schemes [9], the gluon co-efficient is $C_g^1(\alpha_s) = -\frac{\alpha_s}{4\pi} + O(\alpha_s^2)$, while $C_{\mathrm{NS}}^1(\alpha_s) = C_{\mathrm{S}}^1(\alpha_s) = 1 - \frac{\alpha_s}{\pi} + O(\alpha_s^2)$, and we may identify [12]

$$a_0(t) = \Delta q - 2n_f \frac{\alpha_s}{4\pi} \Delta g(t). \tag{41}$$

Thus, the parton model interpretation of the axial current is that it measures both quark spin and gluon spin. The existence of schemes where this identification holds to all orders is assured by the Adler-Bardeen theorem [13]. Δq is scale-independent to all orders, while the scaling of $\Delta g(t)$ is determined by the scaling of the axial current.

Now we make the parton model identifications according to the AB scheme interpretation:

$$\begin{aligned} a_0 &= \Delta q - 2n_f \frac{\alpha_s}{4\pi} \Delta g, \\ B_q(0) &= \frac{1}{2} \Delta q + L_q, \\ B_g(0) &= \Delta g + L_g. \end{aligned} \tag{42}$$

The one-loop mixing derived in (34) then gives us

$$\frac{d}{dt} \Delta q = 0$$

$$\begin{aligned}\frac{d}{dt}L_q(t) &= \frac{\alpha_s}{4\pi}\left[-\frac{8}{3}C_F(\frac{1}{2}\Delta q+L_q)+\frac{2}{3}n_f(\Delta g+L_g)\right]\\ \frac{d}{dt}(L_g(t)+\Delta g(t)) &= \frac{\alpha_s}{4\pi}\left[\frac{8}{3}C_F(\frac{1}{2}\Delta q+L_q)-\frac{2}{3}n_f(\Delta g+L_g)\right]\end{aligned} \tag{43}$$

Thus $d\Delta g/dt$ remains undetermined. For this we require the two-loop evolution of a^0 [15],

$$\frac{d}{dt}a^0(t)=\gamma a^0(t), \qquad\qquad \gamma=-n_f\frac{\alpha_s^2}{2\pi^2}. \tag{44}$$

With the identification (41), this implies

$$\frac{d}{dt}\Delta g(t)=\frac{\alpha_s}{4\pi}\left[3C_F\Delta q+\beta_0\Delta g\right]. \tag{45}$$

Combining (43) and (45), we have the evolution equation, in agreement with ref. [2]:

$$\frac{d}{dt}\begin{pmatrix}\Delta q\\ \Delta g\\ L_q\\ L_g\end{pmatrix}=\frac{\alpha_s}{4\pi}\begin{pmatrix}0 & 0 & 0 & 0\\ 3C_F & \beta_0 & 0 & 0\\ -\frac{4}{3}C_F & \frac{2}{3}n_f & -\frac{8}{3}C_F & \frac{2}{3}n_f\\ -\frac{5}{3}C_F & -11 & \frac{8}{3}C_F & -\frac{2}{3}n_f\end{pmatrix}\begin{pmatrix}\Delta q\\ \Delta g\\ L_q\\ L_g\end{pmatrix}, \tag{46}$$

with

$$\frac{d}{dt}\left(\frac{1}{2}\Delta q+\Delta g+L_q+L_g\right)=0. \tag{47}$$

The general solution to these coupled equations is obtained in ref. [2], whence in the asymptotic limit $t\to\infty$ the partitioning of total quark and gluon angular momentum is $\frac{1}{2}\Delta q+L_q:\Delta g+L_g=3n_f:16$. That result is a direct consequence of how the form factors $B_{q/g}(0)$ mix only with each other. This is the same partitioning between quarks and gluons of the first moments of the unpolarised PDFs.

6 Measurement of Off-Forward Matrix Elements

As was remarked above, of the three form-factors of operators appearing in the angular momentum sum-rule, only one, the axial charge, can be measured in polarised DIS. To measure the other two, one requires off-forward matrix elements of $O_3^{\mu\nu}$ and $O_4^{\mu\nu}$. As has been proposed in ref. [7, 10], these can be extracted from a class of hard, exclusive processes, generically $\gamma^*p\to Xp$. For large Q^2, these factorise into perturbatively calculable pieces, skewed PDFs and other non-perturbative pieces depending on the choice of the final-state species X. The skewed PDFs can be represented as off-forward proton matrix elements of a time-ordered bilocal operator product. The first moments of these are then equal to off-forward matrix elements of local composite operators. Factorisation theorems have been treated for Deeply Virtual Compton Scattering (DVCS) [16, 17, 18],

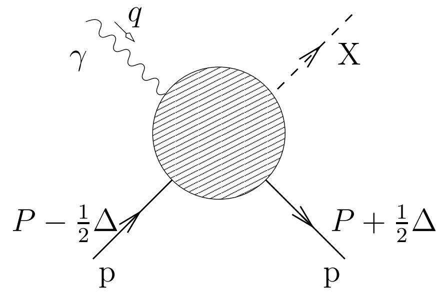


Figure 1: Definitions of four-momenta for hard, exclusive processes.

$\gamma^* p \to \gamma p$, where the final-state photon is real, and also for meson production $\gamma^* p \to pV$. For completeness, we review this work briefly, pointing out that we can measure proton spin components in the combinations $B_q(0) = J_q$ and $B_g(0) = J_g$ [7, 10]. This supplements the isolation of Δq and Δg in polarised DIS and of Δg in open charm production.

Consider then the process $\gamma^* p \to Xp$, where we leave the species X unspecified. The amplitude $\mathcal{M}_X$ for the process factorises as follows:

$$\mathcal{M}_X(q, P, \Delta) \xrightarrow{Q^2 \to \infty} \sum_i \sum_\sigma \int du\, C^i_\sigma(Q^2, \frac{x}{u}, \xi) f_{i/\mathrm{p}}(u, \xi, \Delta^2) D_{\mathrm{X}/\sigma}. \tag{48}$$

The four-momenta are defined as in fig. 1. The sum over i is over partons originating in the proton p; that over σ is for, say, degrees of freedom going into the species X. The Lorentz-invariant kinematic variables are

$$\begin{aligned} Q^2 &= -q^2, & (49)\\ x &= \frac{Q^2}{2q \cdot P}, & (50)\\ \xi &= \frac{q \cdot \Delta}{2q \cdot P} \xrightarrow{Q^2 \to \infty} \frac{\Delta^+}{2P^+}. & (51) \end{aligned}$$

The unpolarised skewed PDFs, which we only need to consider, are given by

$$\begin{aligned} f_{\mathrm{q/p}}(u, \xi, \Delta) &= \int \frac{dy^-}{(2\pi)} e^{-i(u+\xi/2)P^+ y^-} \left\langle P + \frac{1}{2}\Delta \left| T(\overline{\psi}(y^-)\gamma^+ \mathcal{P}(y^-, 0)\psi(0)) \right| P - \frac{1}{2}\Delta \right\rangle, \\ P^+ f_{\mathrm{g/p}}(u, \xi, \Delta) &= \int \frac{dy^-}{(2\pi)} e^{-i(u+\xi/2)P^+ y^-} \left\langle P + \frac{1}{2}\Delta \left| T(F^{+\alpha}(y^-)\mathcal{P}(y^-, 0)F_\alpha{}^+(0)) \right| P - \frac{1}{2}\Delta \right\rangle, \end{aligned} \tag{52}$$

where $\mathcal{P}(x, y)$ is the path-ordered string operator which renders a bilocal operator depending on two points x and y gauge-invariant:

$$\mathcal{P}(x, y) = P \exp\left(-ig \int_x^y ds \cdot A\right). \tag{53}$$

In the forward limit, $\Delta \to 0$, these tend toward the usual PDFs with the correspondence

$$\begin{aligned} f_{\mathrm{q/p}}(u,\xi,\Delta) &\stackrel{\Delta\to 0}{\longrightarrow} f_{\mathrm{q/p}}(u), \\ f_{\mathrm{g/p}}(u,\xi,\Delta) &\stackrel{\Delta\to 0}{\longrightarrow} u\, f_{\mathrm{g/p}}(u). \end{aligned} \tag{54}$$

The skewed PDFs have support for $-1 < u < 1$, so their first moments w.r.t. u are

$$\begin{aligned} (P^+)^2 \int_{-1}^{1} du\, u\, f_{\mathrm{q/p}}(u,\xi,\Delta) &= \left\langle P + \frac{1}{2}\Delta \left| \overline{\psi}\gamma^+ i \overset{\leftrightarrow}{D}{}^+ \psi(0) \right| P - \frac{1}{2}\Delta \right\rangle, \\ (P^+)^2 \int_{-1}^{1} du\, f_{\mathrm{q/p}}(u,\xi,\Delta) &= \left\langle P + \frac{1}{2}\Delta \left| F^{+\alpha} F_{\alpha}{}^{+}(0) \right| P - \frac{1}{2}\Delta \right\rangle. \end{aligned} \tag{55}$$

Decomposing the matrix elements as in section 3 we can relate the form factors at zero momentum transfer $B_q(0)$ and $B_g(0)$ to the first derivatives of the first moments of the PDFs:

$$-iP^+ \frac{\partial}{\partial \Delta_\mu} \int_{-1}^{1} du\, u\, f_{\mathrm{q/p}}(u,\xi,\Delta)\Big|_{\Delta=0} = \frac{B_q(0)}{M} \epsilon^{+\mu\alpha\beta} P_\alpha s_\beta, \tag{56}$$

$$-iP^+ \frac{\partial}{\partial \Delta_\mu} \int_{-1}^{1} du\, f_{\mathrm{g/p}}(u,\xi,\Delta)\Big|_{\Delta=0} = \frac{B_g(0)}{M} \epsilon^{+\mu\alpha\beta} P_\alpha s_\beta, \tag{57}$$

where

$$B_q(0) = J_q, \tag{58}$$

$$B_g(0) = J_g. \tag{59}$$

Thus, if the spin vector s^μ-dependent part of the first derivative of the first moments of the skewed PDFs can be extracted from experiment, then this gives information on the total quark and total gluon angular momentum.

7 Gauge-Non-Invariant Decompositions

Here we discuss further the gauge-non-invariant decomposition of the AMC of eq. (8), in particular the possible merits of relating this to the four components Δq, Δg, L_q and L_g in one particular choice of gauge and Lorentz frame. The natural choice for a parton model interpretation would be the axial gauge $A^+ = 0$ and the infinite momentum frame, that is, the rest frame of the virtual photon in DIS in the limit $Q^2 \to \infty$. Indeed, one finds attempts [19] to relate moments of angular momentum PDFs, $\Delta q(x)$, $\Delta g(x)$, $L_q(x)$ and $L_g(x)$ to local, composite operators in the gauge $A^+ = 0$. However, one is henceforth restricted to a particular choice of gauge and quantisation procedure, viz. light-cone quantisation, which is inconvenient beyond low orders in perturbation theory. We also

discuss attempts to relate Δg to the forward matrix elements of the Chern-Simons current k^μ, and review the problem which befalls this identification, namely the coupling of k^μ to unphysical pseudoscalar poles.

To begin, suppose that we quantise QCD in the infinite momentum frame with the gauge $A^+ = 0$, i.e. use light-cone quantisation. In this picture, the only propagating degrees of freedom are the transverse gluon polarisations, $A^i_\perp$, and we do not require any gauge-fixing fields or ghosts. Since the fields are quantised with equal x^+ commutation relations,[5] then the canonical rotation generator is

$$J^{ij}_{(+)} = \int dx^- \, d^2x^\perp \, M^{+ij}, \tag{60}$$

with

$$M^{\mu\nu\lambda} = \frac{1}{2}\epsilon^{\mu\nu\lambda\alpha}\overline{\psi}\gamma_\alpha\gamma^5\psi + i\overline{\psi}\gamma^\mu x^{[\nu}\partial^{\lambda]}\psi - F^{\mu[\nu}A^{\lambda]} - F^{\mu\alpha}x^{[\nu}\partial^{\lambda]}A_\alpha. \tag{61}$$

Thus,

$$\begin{aligned} J^{ij}_{(+)} &= \int dx^- \, d^2x^\perp \, \Big[\sqrt{2}\psi^\dagger_+\sigma^{ij}\psi_+ + i\sqrt{2}\psi^\dagger_+ x^{[i}\partial^{j]}\psi_+ \\ &- \partial^+ A^{[i}A^{j]} + \partial^+ A^k_\perp x^{[i}\partial^{j]}A^k_\perp\Big], \end{aligned} \tag{62}$$

where $\psi_+ = \Lambda^+\psi = \frac{1}{\sqrt{2}}\gamma^0\gamma^+\psi$. This equation is of the same form as eq. (11). The difficulty here now is that the four operators are not even invariant under the residual gauge transformations which leave the condition $A^+ = 0$ unchanged. In ref. [19], however, definitions of $\Delta q(x)$, $\Delta g(x)$, $L_q(x)$ and $L_g(x)$ are constructed in the axial gauge which are invariant under residual gauge transformations. The first moments of these, $\int_{-1}^1 \Delta q(x)dx$ etc., correspond to the four operators in eq. (62), but with $x^{[i}\partial^{j]}$ replaced by $x^{[i}\mathcal{D}^{j]}$, with $\mathcal{D}^\mu$ being the derivative covariant under the residual gauge transformation. It is not altogether clear that those modified operators add up to give a conserved current. Moreover, one is restricted to a particular gauge, so if one encounters technical difficulties in perturbative calculations of the different contributions to the proton spin, as reported in ref. [5], one has no room for manoeuvre.

The other feature of axial gauges is that the $(+12)$ component of $F^{\mu[\nu}A^{\lambda]}$ is related to the Chern-Simons current k^μ.

$$k^\mu = \frac{\alpha_s}{2\pi}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left[A_\nu\partial_\rho A_\sigma - \frac{2ig}{3}A_\nu A_\rho A_\sigma\right]. \tag{63}$$

Indeed, putting $A^+ = 0$, we have $k^+ = \frac{\alpha_s}{4\pi}\partial^+ A^{[1}A^{2]}$. Note that this is a frame-dependent and gauge-dependent statement. Some authors [12, 20, 21] have tried to identify the forward matrix element of k^μ with $-\frac{\alpha_s}{4\pi}\Delta g$ in such a way that Δg is at least a Lorentz scalar form factor: From the anomalous Ward identity, which is not renormalised,

$$\partial_\mu j^\mu_5 - 2n_f Q \simeq 0, \tag{64}$$

[5] Our convention is that $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$.

with $Q = \frac{\alpha_s}{8\pi}\mathrm{tr}F^{\mu\nu}F_{\mu\nu} = \partial_\mu k^\mu$ and $j_5^\mu = \overline{\psi}\gamma^\mu\gamma_5\psi$, one deduces that $j_5^\mu - 2n_f k^\mu$ is not renormalised, so that with

$$\begin{aligned} \langle p, s\,|k^\mu|\,p', s\rangle &= 2Ms^\mu a_g^0(\Delta^2) + s\cdot\Delta\Delta^\mu p_g(\Delta^2), \\ \langle p, s\,|(j_5^\mu - 2n_f k^\mu)|\,p', s\rangle &= 2Ms^\mu a_q^0(\Delta^2) + s\cdot\Delta\Delta^\mu p_q(\Delta^2), \end{aligned} \tag{65}$$

we have

$$\langle p, s\,|j_5^\mu|\,p, s\rangle = 2Ms^\mu(a_q^0(0) + 2n_f a_g^0(0)), \tag{66}$$

so that

$$\begin{aligned} \Delta q &= a_q^0(0) \\ -\frac{\alpha_s}{4\pi}\Delta g &= a_g^0(0). \end{aligned} \tag{67}$$

This is true provided, as should be the case, that $\lim_{\Delta\to 0}[2n_f p_g(\Delta^2) + p_q(\Delta^2)]$ is finite.

However, in covariant gauges, the presence of an unphysical massless pseudoscalar pole [22] coupling to k^μ means that as $\Delta \to 0$

$$\begin{aligned} 2n_f p_g(\Delta^2) &\to \frac{1}{\Delta^2}, \\ p_q(\Delta^2) &\to -\frac{1}{\Delta^2}. \end{aligned} \tag{68}$$

In other words, the forward matrix element of the Chern-Simons current is singular, when treated non-perturbatively. In axial gauges, $n\cdot A = 0$, this pole becomes a $\frac{1}{n\cdot\Delta}$ pole, so that the Lorentz structure $n^\mu/(n\cdot\Delta)$ is present instead of $\Delta^\mu/(\Delta^2)$ in the limit $\Delta \to 0$. This has been shown for the Schwinger model explicitly in ref. [23] and for QCD in ref. [24]. That said, for the gauge $A^+ = 0$, $n^+ = 0$, so no such pole is present in $\langle p, s\,|k^+|\,p, s\rangle$; only for one choice of gauge and frame can the forward matrix element be well defined.

In conclusion, defining observables in terms of matrix elements of gauge-non-invariant operators requires reference to a specific gauge, which is problematic. The matrix elements are also singular in the forward limit. Since the gauge-non-invariant approach is sufficient for our purposes, there is no need for further decomposition of the AMC.

8 Discussion and Conclusions

In summary, we have decomposed the conserved angular momentum current keeping it gauge invariant as far as possible. We have then decomposed the forward matrix elements of these operators into scalar form factors, and shown that the axial charge drops out of the angular momentum sum-rule. This is important because it provides evidence that the axial charge is not straightforwardly interpreted as a measure of parton intrinsic spin.

Indeed, the parton model interpretation which we have discussed in section 5 is such that Δq and Δg appear in form factors multiplying two different Lorentz structures, one of which cancels out of the sum-rule. One should thus take the view that although it is meaningful to split angular momentum into quark and gluon pieces, a further split into spin and orbital pieces is frame dependent and therefore not a property of the proton itself.

We have derived the mixing of the three gauge-invariant operators in the AMC, and calculated this explicitly to one loop. In general, we have shown how the axial anomaly cannot affect the renormalisation of the AMC, and this is reflected at the level of form factors by the decoupling of the axial charge from the angular momentum sum rule.

Despite having argued that it is unnatural to do so, we have also derived the mixing of Δq, Δg, L_q and L_g by relating these to form factors of the gauge-invariant composite operators, and found results in agreement with what is obtained by taking the first moments of splitting functions. The scaling of Δg has been determined from the scaling of the axial charge provided Δq is constrained to be scale-independent. This has enabled us to manage with just three operators. Thus, the mixing matrix of HJT can be derived in an explicitly gauge-invariant way in terms of local, composite operators. This means that we should not need to consider using gauge-non-invariant operators.

Appendix A

In this appendix we derive an expression for the forward matrix element of an operator containing a factor x^μ. This must be done with care otherwise we end up with a derivative of the momentum-conserving delta-function. To see this, consider the off-forward nucleon matrix element of an operator $x^{[i}B^{j]0}$:

$$\begin{aligned}\int \mathrm{d}^3x \left\langle p \left| x^{[i} B^{j]0}(x) \right| p' \right\rangle &= \int \mathrm{d}^3x e^{i(p'-p)\cdot x} x^{[i} \left\langle p \left| B^{j]0}(0) \right| p' \right\rangle \\ &= i(2\pi)^3 \frac{\partial}{\partial \Delta_{[i}} \left(\delta^3(p-p') \right) \left\langle p \left| B^{j]0}(0) \right| p' \right\rangle , \qquad (69)\end{aligned}$$

where $\langle p\,|B(0)|\,p'\rangle$ is a Green function amputated w.r.t. its p and p' legs and where P is held constant in the derivative. We may fix this by replacing $|p'\rangle$ with a wavepacket,

$$|\Phi\rangle = \int \frac{d^3\Delta}{(2\pi)^3} \Phi(p')\, |p'\rangle , \qquad (70)$$

such that $\Phi(p')$ has no azimuthal dependence about the k-direction, perpendicular to the i- and j-directions. Then, integrating by parts, we have

$$\int \mathrm{d}^3x \left\langle p \left| x^{[i} B^{j]0}(x) \right| \Phi \right\rangle = -i \frac{\partial}{\partial \Delta_{[i}} (\Phi(p')) \Big|_{p'=p} \left\langle p \left| B^{j]0}(0) \right| p \right\rangle$$

$$- \quad \Phi(p)\, i\frac{\partial}{\partial \Delta_{[i}} \left\langle p \left| B^{j]0}(0) \right| p' \right\rangle \Big|_{p'=p} . \tag{71}$$

The first term on the r.h.s. vanishes since $\langle p\,|B^{0j}(0)|\,p\rangle = p^0 p^j \left\langle p \left| \widetilde{B} \right| p \right\rangle$, with $\left\langle p \left| \widetilde{B} \right| p \right\rangle$ the reduced matrix element, so that we get a factor $p^{[j} \frac{\partial}{\partial p_{i]}} \Phi(p)$ which vanishes by construction — i.e. the wavepacket carries no OAM. Thus we consider only the term

$$\Phi^{-1}(p) \int \mathrm{d}^3x \left\langle p \left| x^{[i} B^{j]0}(x) \right| \Phi \right\rangle = -i\frac{\partial}{\partial \Delta_{[i}} \left\langle p \left| B^{j]0}(0) \right| p' \right\rangle \Big|_{p'=p} . \tag{72}$$

Notice that the l.h.s. reduces to $\langle p\,|B(0)|\,p\rangle$ if we remove the factor x^μ and talk about the matrix element of an ordinary, local, composite operator.

Next, we show that matrix elements of operators such as $\partial^\alpha(x^\beta B(x))$ vanish, so that the forward matrix element of the total derivative term in eq. (5) can be neglected. We get

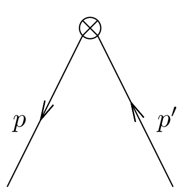
$$\begin{aligned} \int \mathrm{d}^3x \left\langle p \left| \partial^\alpha (x^\beta B(x)) \right| \Phi \right\rangle \;&=\; -(2\pi)^3 \int \frac{d^3\Delta}{(2\pi)^3} (p-p')^\alpha \delta^3(p-p') \left[\frac{\partial}{\partial \Delta_\beta} (\Phi(p')) \left\langle p\,|B(0)|\,p' \right\rangle \right. \\ &\left. +\quad \Phi(p') \frac{\partial}{\partial \Delta_\beta} \left\langle p\,|B(0)|\,p' \right\rangle \right], \end{aligned} \tag{73}$$

which is zero.

Appendix B

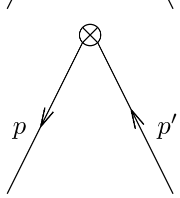
To evaluate the UV divergent pieces to one loop, we insert the operators $O_{1\to4}$ into Green functions with either quark/anti-quark legs or two gluon legs, and calculate all the diagrams in figs. 2 and 3. The bare vertices, symbolised by $\otimes$, have the following Feynman rules, which also give the tree-level diagrams:

$O_1^{\mu\nu\lambda}$:



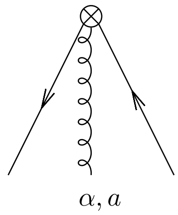
$$\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\gamma_\sigma\gamma^5 \tag{74}$$

$O_2^{\mu\nu}$:



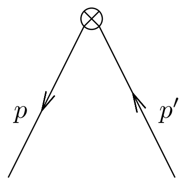
$$\gamma^\mu P^\nu \tag{75}$$

&



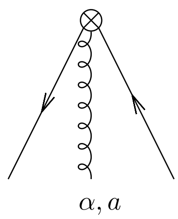
$$g\gamma^{\mu}g^{\nu}_{\alpha}t^{a} \tag{76}$$

$O_4^{\mu\nu}$:



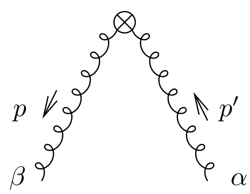
$$\frac{1}{2}\gamma^{\{\mu}P^{\nu\}} \tag{77}$$

&



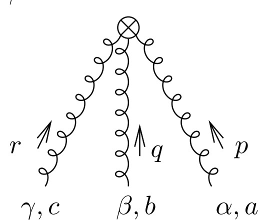
$$\frac{1}{2}\gamma^{\{\mu}g^{\nu\}}_{\alpha}t^{a} \tag{78}$$

$O_3^{\mu\nu}$:



$$\begin{aligned}&-p^{\{\mu}p'^{\nu\}}g_{\alpha\beta} - p\cdot p' g^{\mu}_{\{\alpha}g^{\nu}_{\beta\}}\\ &\quad +p^{\{\mu}g^{\nu\}}_{\alpha}p'_{\beta} \ + p'^{\{\mu}g^{\nu\}}_{\beta}p_{\alpha}\end{aligned} \tag{79}$$

&



$$\begin{aligned}-igf^{abc}\Big[&p^{\{\mu}g^{\nu\}[\gamma}g^{\beta]\alpha} - g^{\alpha\{\mu}g^{\nu\}[\gamma}p^{\beta]}\\ &q^{\{\mu}g^{\nu\}[\alpha}g^{\gamma]\beta} - g^{\beta\{\mu}g^{\nu\}[\alpha}q^{\gamma]}\\ &r^{\{\mu}g^{\nu\}[\beta}g^{\alpha]\gamma} - g^{\gamma\{\mu}g^{\nu\}[\beta}r^{\alpha]}\Big]\end{aligned} \tag{80}$$

where $P = \frac{1}{2}(p + p')$, $\Delta = p - p'$.

We evaluate the $\frac{1}{\epsilon}$ pole pieces of the Feynman diagrams and keep only terms of order Δ. For example, diagram 2a with $\otimes = O_2$ gives us, in the Feynman gauge:

$$\begin{aligned}&C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\left[-\frac{1}{3}\gamma^{\{\mu}P^{\nu\}} + \gamma^{\mu}P^{\nu} - \frac{1}{3}g^{\mu\nu}\not{P} - \frac{1}{2}i\Delta_{\alpha}\epsilon^{\mu\nu\alpha\sigma}\gamma_{\sigma}\gamma^{5}\right]\\ &= \ C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\left[-\frac{2}{3}\left\langle q\left|O_4^{\mu\nu}\right|q\right\rangle_{\text{tree}} + \left\langle q\left|O_2^{\mu\nu}\right|q\right\rangle_{\text{tree}} + \frac{1}{3}g^{\mu\nu}\left\langle q\left|E_1\right|q\right\rangle_{\text{tree}} - \left\langle q\left|\partial_{\alpha}O_1^{\mu\nu\alpha}\right|q\right\rangle_{\text{tree}}\right].\end{aligned} \tag{81}$$

Adding up the diagrams in figs. 2 and 3 and their left-right mirror images, where appropriate, we get, neglecting $O(\epsilon^0)$, $O(\Delta^2)$ and $g^{\mu\nu}$ terms,

$$\left\langle q\left|O_1^{\mu\nu\lambda}\right|q\right\rangle_B \ = \ \left(1 - C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\right)\left\langle q\left|O_1^{\mu\nu\lambda}\right|q\right\rangle_{\text{tree}} \tag{82}$$

$$
\begin{aligned}
\langle q\,|O_2^{\mu\nu}|\,q\rangle_B &= -\frac{8}{3}C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\langle q\,|O_4^{\mu\nu}|\,q\rangle_{\text{tree}} \\
&\quad + \left(1 - C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\right)\langle q\,|O_2^{\mu\nu}|\,q\rangle_{\text{tree}} && (83) \\
\langle q\,|O_3^{\mu\nu}|\,q\rangle_B &= \frac{8}{3}C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\langle q\,|O_4^{\mu\nu}|\,q\rangle_{\text{tree}} && (84) \\
\langle q\,|O_4^{\mu\nu}|\,q\rangle_B &= \left(1 - \frac{11}{3}C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\right)\langle q\,|O_4^{\mu\nu}|\,q\rangle_{\text{tree}} && (85) \\
\left\langle g\left|O_1^{\mu\nu\lambda}\right|g\right\rangle_B &= \text{finite} && (86) \\
\langle g\,|O_2^{\mu\nu}|\,g\rangle_B &= \frac{2}{3}n_f\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\langle g\,|O_3^{\mu\nu}|\,g\rangle_{\text{tree}} && (87) \\
\langle g\,|O_3^{\mu\nu}|\,g\rangle_B &= \left(1 + \left(\frac{5}{3} - \frac{4}{3}n_f\right)\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\right)\langle g\,|O_3^{\mu\nu}|\,g\rangle_{\text{tree}} && (88) \\
\langle g\,|O_4^{\mu\nu}|\,g\rangle_B &= \langle g\,|O_2^{\mu\nu}|\,g\rangle_B\,. && (89)
\end{aligned}
$$

Some of these poles come from the field-strength divergences of figs. 2b and 3e-h and are absorbed by multiplying each quark and gluon external leg by $Z_\psi^{1/2}$ and $Z_A^{1/2}$ respectively, with

$$
\begin{aligned}
Z_A &= \left(1 - \left(\frac{5}{3}C_F - \frac{2}{3}n_f\right)\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\right), \\
Z_\psi &= \left(1 + C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\right). && (90)
\end{aligned}
$$

Writing

$$
\begin{aligned}
\langle q\,|O_B|\,q\rangle_R &= Z_\psi\,\langle q\,|O_B|\,q\rangle_B\,, \\
\langle g\,|O_B|\,g\rangle_R &= Z_A\,\langle g\,|O_B|\,g\rangle_B\,, && (91)
\end{aligned}
$$

we have

$$
\begin{aligned}
\langle q\,|O_{1,B}|\,q\rangle_R &= \langle q\,|O_{1,B}|\,q\rangle_{\text{tree}} && (92) \\
\langle q\,|O_{2,B}|\,q\rangle_R &= \langle q\,|O_{2,B}|\,q\rangle_{\text{tree}} - \frac{8}{3}C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\langle q\,|O_{4,B}|\,q\rangle_{\text{tree}} && (93) \\
\langle q\,|O_{3,B}|\,q\rangle_R &= \frac{8}{3}C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\langle q\,|O_{4,B}|\,q\rangle_{\text{tree}} && (94) \\
\langle q\,|O_{4,B}|\,q\rangle_R &= \left(1 - \frac{8}{3}C_F\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\right)\langle q\,|O_{4,B}|\,q\rangle_{\text{tree}} && (95) \\
\langle g\,|O_{1,B}|\,g\rangle_R &= \text{finite} && (96) \\
\langle g\,|O_{2,B}|\,g\rangle_R &= \frac{2}{3}n_f\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\langle g\,|O_{3,B}|\,g\rangle_{\text{tree}} && (97)
\end{aligned}
$$

$$\langle g\,|O_{3,B}|\,g\rangle_R = \left(1 - \frac{2}{3}n_f\frac{1}{\epsilon}\frac{g^2}{(4\pi)^2}\right)\langle g\,|O_{3,B}|\,g\rangle_{\rm tree} \tag{98}$$

$$\langle g\,|O_{4,B}|\,g\rangle_R = \langle g\,|O_{2,B}|\,g\rangle_R\,. \tag{99}$$

The renormalisation matrix required to remove these remaining poles is then given by eq. (31).

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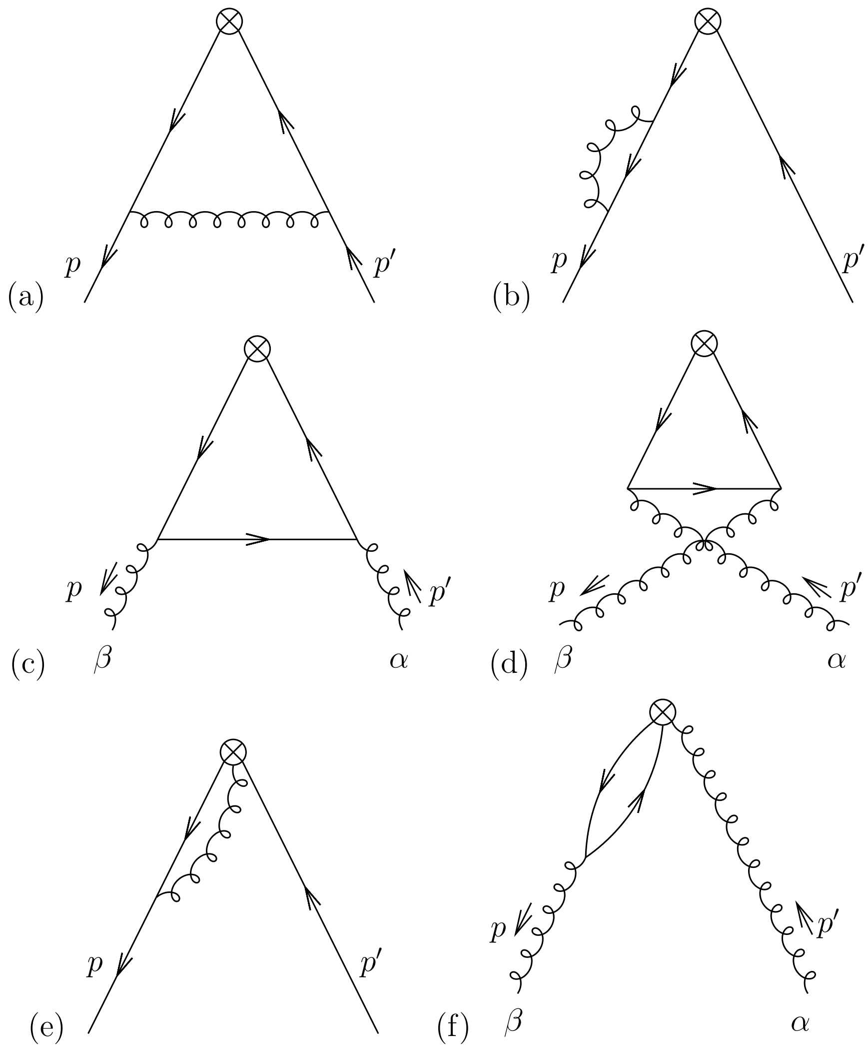

Figure 2: One-loop diagrams

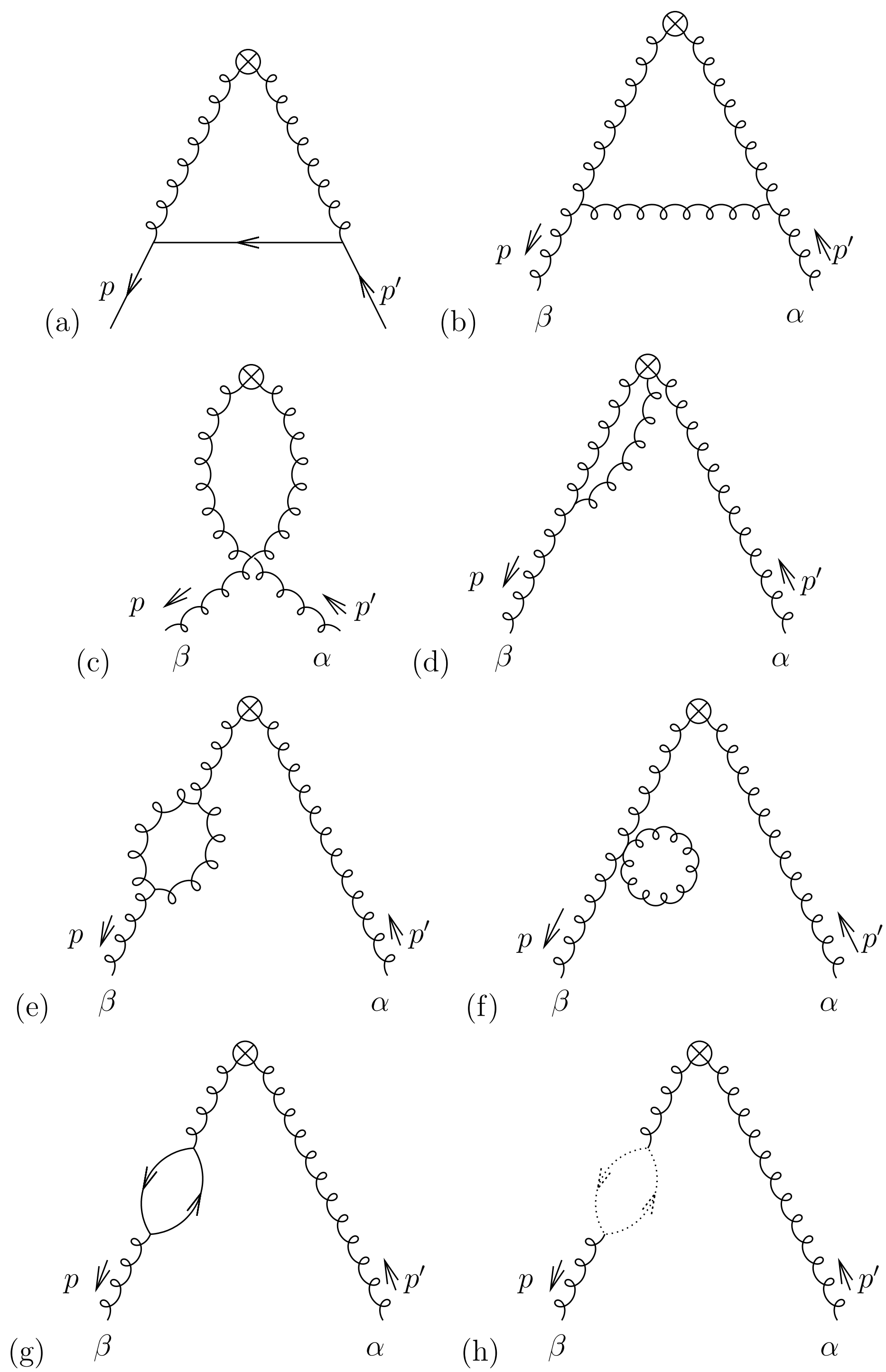

Figure 3: One-loop diagrams